



Existence and Uniqueness for a One-Dimensional Transmission Problem with Thermal Coupling and Delayed Velocity Feedback

Aissa Benseghir⁽¹⁾ and Mourad Dilmi⁽²⁾

⁽¹⁾*Setif 1 University Ferhat Abbas, Department of Mathematics, Setif, Algeria
e-mail: aissa.benseghir@univ-setif.dz*

⁽²⁾*Setif 1 University Ferhat Abbas, Department of Mathematics, Setif, Algeria
e-mail: mourad.dilmi@univ-setif.dz*

Abstract

This work addresses the well-posedness of a one-dimensional transmission problem involving a wave equation with both thermal coupling (Fourier model) and a time-delay term acting on the velocity. The physical domain consists of two elastic regions separated by an intermediate elastic medium, with transmission conditions at the interfaces. The delay effect is incorporated via a past-history term, and thermal effects are modeled through a coupled heat equation. Assuming positive physical parameters and a smallness condition on the delayed damping coefficient, we reformulate the system as an abstract Cauchy problem in a suitable Hilbert space by introducing an auxiliary variable for the delay history. Using semigroup theory, we establish existence and uniqueness results for mild and strong solutions, depending on the regularity of the initial data.

Keywords: Wave equation, transmission problem, delay term, exponential stability, past-history term.

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