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The Discrete Meijer G-Function

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Abstract

The classical Meijer G-function provides a unified representation for a broad class of special functions and plays an important role in the theory of differential equations. In this work, we introduce a discrete analogue of the Meijer G-function based on a Mellin-Barnes type contour integral adapted to finite-difference calculus. The classical power function is replaced by generalized falling factorials, which naturally arise in the discrete setting.

We establish fundamental properties of the discrete Meijer G-function, including parameter-shift formulas, recurrence relations, and operational identities involving finite-difference analogues of the Euler operator. We derive a linear difference equation satisfied by the discrete Meijer G-function and prove its connection with the classical Meijer G-function.

Furthermore, we show that many discrete special functions admit representations in terms of the proposed function. In particular, we obtain representations for discrete hypergeometric functions, discrete trigonometric and Bessel functions, discrete Chebyshev and Legendre polynomials, and discrete polylogarithm functions. The results suggest that the discrete Meijer G-function may serve as a universal special function in finite-difference calculus and provide a useful framework for the study of difference equations and discrete special functions.

Keywords: Meijer G-function, hypergeometric function, Mellin-Barnes integral.

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