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Approximation by Neural Network Operators of Convolution Type Activated by Deformed and Parametrized Half Hyperbolic Tangent Function

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Abstract

The idea behind neural networks and artificial neural networks is to act like human brain by understanding the biological nature and function of it and they lie in the centre of machine learning. Recently, the power of computers, accessing the big data, studies on neural networks gain speed since they give us opportunity to handle some complex duties such as image recognition, natural language processing, optimization, forecasting, approximation of functions and curve fitting. Quantitative approximation of positive linear operators to the unit has been studied by G. A. Anastassiou and many mathematicians. In the present study, we introduce three kinds of convolution neural network operators with the kernel based on symmetrized, q-deformed and β-parametrized half hyperbolic tangent function. It is noteworthy to mention that it is frequently used in neural networks and they can be interpreted as positive linear operators, thus we use the methods of positive linear operator theory. We obtain quantitative convergence results to the identity operator by using modulus of continuity and global smoothness preservation of our operators are also examined. Furthermore, iterated versions of them are taken into consideration.

Keywords: Half hyperbolic tangent function, convolution type, positive linear operators, rate of convergence, iterated approximation.

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