



# Uniqueness and Ulam Stability Results for a Coupled System of Fuzzy Fractional Differential Equations via the $\chi$ -Fuzzy Fractional Derivative

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## Abstract

The concept of fuzzy set theory was developed by Lotfi Zadeh (L.A, 1965 [2]) as a method for handling incomplete information across a wide range of applications in various fields. Notably, this concept differs from ordinary set theory due to the numerous classes of functions and symbols it involves. Consequently, the outcomes related to fractional differential equations [1] and fuzzy numbers are more accurate and of higher quality.

This work aims to present novel qualitative results for a coupled system of neutral fuzzy fractional differential equations characterized by generalized  $\chi$ -fuzzy fractional derivative of order  $0 < \nu < 1$ ; an area that remains insufficiently explored and limited in available literature. The system is first reformulated into an equivalent integral representation using the generalized Hukuhara difference  $\ominus_{gH}$ . In order to support the analytical framework, a new metric structure is constructed by incorporating the Mittag-Leffler function within a fuzzy functional setting. Alongside, we apply the Banach contraction principle with fractional calculus ideas as well as fuzzy numbers criterion to ensure the existence and uniqueness of the derived solution. Ultimately, our study takes into account two forms of Ulam stability, establishing new theoretical insights through sound analytical techniques.

**Keywords:**  $\chi$ -Fuzzy fractional differential equations, fixed-point theorem, Ulam stability.

## References:

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- [2] L. A. Zadeh, Fuzzy sets. Information and Control. 8 (1965), 338–35.