



Shearlet Expansions and Asymptotic Behavior of Lizorkin Distributions

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Abstract

While wavelets are effective for detecting the singular support of functions and distributions, their isotropic nature limits their ability to capture finer geometric features of singularities, such as those encoded by the wavefront set. To overcome these limitations and extend the advantages of wavelet methods to higher dimensions, several directional frameworks have been developed. Among them, shearlet systems, introduced by Guo, Kutyniok and Labate, provide an effective approach due to their strong analytical and geometric properties [1, 2, 3].

Following recent developments, the shearlet transform has been extended, via duality, to Lizorkin-type spaces of test functions and distributions, $\mathcal{S}_0(\mathbb{R}^2)$ and $\mathcal{S}'_0(\mathbb{R}^2)$ [4]. In our recent work [5], we develop a shearlet expansion theory in these spaces. We prove that shearlet series associated with Parseval systems converge in the corresponding topologies and obtain a topological characterization of Lizorkin distributions in terms of their shearlet coefficients. This framework also provides descriptions of bounded sets and weakly convergent nets via shearlet coefficients.

Finally, we apply the developed theory to characterize the quasiasymptotic behavior and quasiasymptotic boundedness of Lizorkin distributions through the asymptotic behavior of their shearlet coefficients.

Keywords: Parseval shearlet, shearlet coefficients, discrete shearlet transform, Tauberian theorems, asymptotic behavior, Lizorkin-type spaces of test functions and distributions.

References:

- [1] G. Kutyniok and D. Labate, Shearlets: Multiscale Analysis for Multivariate Data. Appl. Numer. Harmon. Anal., Birkhäuser/Springer, New York, 2012.
- [2] K. Guo and D. Labate, Optimally sparse multidimensional representation using shearlets. SIAM J. Math. Anal. 39 (2007), no. 1, 298–318.
- [3] G. Kutyniok and D. Labate, Resolution of the wavefront set using continuous shearlets. Trans. Amer. Math. Soc. 361 (2009), no. 5, 2719–2754.
- [4] F. Bartolucci, S. Pilipović and N. Teofanov, Continuity properties of the shearlet transform and the shearlet synthesis operator on the Lizorkin type spaces. Math. Nachr. 295 (2022), no. 12, 2318–2337.
- [5] A. R. Ferizi and K. Hadzi-Velkova Saneva, Shearlet expansion theory on Lizorkin-type spaces. Funct. Anal. Appl. 60 (2026), no. 1, 82–96.