



Approximate Formulas for Boundary Functionals of Random Walk Process

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Abstract

In this study, a random walk process is considered and two important boundary functionals of the process, $N(z)$ and $S_{N(z)}$, are investigated. First, using Dynkin's principle together with the ladder times and ladder heights of the random walk process, the boundary functionals $N(z)$ and $S_{N(z)}$ are represented as renewal-reward processes. Then, we use basic methods from renewal theory to derive approximate formulas for the expected value and variances of these boundary functionals. It is demonstrated that the coefficients of the obtained formulas can be expressed in terms of the moments of the first ladder time and the ladder height of the random walk process. Finally, the numerical values of these coefficients are computed for several special cases using the Monte Carlo method, leading to the proposal of novel, practical approximate formulas for the expected values and variances of the boundary functionals.

Keywords: Random walk process, Dynkin's principle, approximate formulas.

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