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A Robust Numerical Framework for the Modified Regularized Long-Wave Equation Using Quartic Trigonometric Tension B-Spline Collocation

Azhar Iqbal

Prince Mohammad Bin Fahd University, Department of Mathematics, Al Khobar, Saudi Arabia
e-mail: aiqbal@pmu.edu.sa

Abstract

The Modified Regularized Long-Wave equation is one of the fundamental nonlinear wave equations describing various physical phenomena, including nonlinear wave propagation in plasma and shallow water. This study presents a high-order numerical scheme based on Quartic Trigonometric Tension B-spline (QT-TB) functions. The spatial discretization is performed using the collocation method, while the Crank–Nicolson scheme is employed for time integration. The proposed QT-TB approach incorporates a tension parameter that enhances spline performance in regions with steep gradients. The nonlinear term is linearized via Taylor series expansion, leading to a computationally efficient tridiagonal system. Stability analysis using the Von Neumann method confirms that the scheme is unconditionally stable. Numerical experiments involving single, double, and triple soliton interactions demonstrate the accuracy and efficiency of the method. The results indicate second-order accuracy in both space and time, along with good conservation of invariants.

Keywords: MRLW equation, trigonometric B-splines, tension parameter, soliton interaction, collocation method, Crank-Nicolson scheme.

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