



# Fixed Point Methods for Nonlinear Boundary Value Problems in Complex Spaces

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## Abstract

Boundary value problems for complex partial differential equations constitute a central topic in modern complex analysis and operator theory, with significant applications in mathematical physics and nonlinear analysis. In this study, we develop a general analytical framework for investigating boundary value problems associated with higher-order complex differential operators in complex domains. The main objective is to establish existence and uniqueness results by employing fixed point techniques in suitable functional spaces.

The proposed approach relies on transforming the original boundary value problem into an equivalent operator equation through integral representations derived from complex analytic methods. By introducing appropriate integral operators acting on complex Banach spaces, the problem is reformulated as a nonlinear operator equation whose solvability can be analyzed using fixed point theory. The structural properties of the associated operators, including boundedness, compactness, and continuity, are carefully examined to ensure the applicability of contraction-type principles.

Under general assumptions such as Lipschitz continuity and suitable growth conditions on the nonlinear terms, we prove the existence of solutions via the Banach fixed point theorem and related fixed point results. The framework is formulated in a sufficiently general setting, allowing its application to a broad class of boundary value problems without restriction to a specific boundary condition or geometry.

The results provide a unified functional-analytic perspective on complex boundary value problems and highlight the effectiveness of fixed point methods in complex spaces. This approach offers a flexible basis for further extensions to nonlinear higher-order equations, generalized analytic functions, and operator-theoretic formulations in complex analysis.

**Keywords:** Complex boundary value problems, fixed point theory, nonlinear complex equations.

## References:

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