



Solution of Continuous-Time Stochastic Optimal Control Problems Using Euler-Maruyama Method via Pontryagin's Principle

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Abstract

In this work, a continuous-time deterministic optimal control problem in [1] was randomized by incorporation of stochastic process to the necessary conditions obtained via Pontryagin's maximum principle and solved using Euler-Maruyama method [2]. The solution was obtained by considering volatility rate ($\sigma = 0$ and $\sigma > 0$) where at $\sigma = 0$, the solution is completely deterministic, while at $\sigma > 0$, the solution exhibits random effects which consequently gives a stochastic optimal value. The effectiveness of the procedure is tested by solving three examples (maximization and minimization problems), where the results are tabulated and plotted on graphs. Stability analysis was also performed, and the procedure proved to be effective such that for a given boundary condition, a better optimal value could be attained at a certain volatility rate. Thus, the time path for the state variable gives the time path for the objective function.

Keywords: Optimal control, stochastic, continuous-time.

References:

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- [2] F. Flandoli, F. Russo and J. Wolf, Some SDEs with distributional drift. I. *General calculus. Osaka J. Math.* 40 (2003), no. 2, 493–542.