Polynomial and Product Integration Approximations for Volterra Integral Equations with Tempered Kernels

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Abstract

In this study, the numerical solution of second-kind Volterra integral equations whose kernels combine an algebraic weak singularity with an exponential tempering factor are studied. This structure arises naturally in anomalous diffusion and fading-memory models.

Two complementary methods are constructed and compared. The first, called Exponential-Tempered Product Integration, evaluates all singular-exponential quadrature weights exactly in closed form via the tempered incomplete gamma function. The second, Generalized Laguerre Spectral Collocation, expands the solution in a basis of weighted Laguerre polynomials that absorbs both the algebraic singularity and the exponential decay analytically. A detailed comparison in terms of error, condition number, and CPU time shows that Exponential-Tempered Product Integration method is approximately five times faster and produces a near-perfectly conditioned system, making it the more robust choice for practical computation.

Keywords: Volterra integral equation, tempered weakly singular kernel, product integration, incomplete gamma function.

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