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2D Nonstationary Wavelet Filter Design via Bézout and Sum-of-Squares Decomposition

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Abstract

In this work, we propose a framework for the design of two-dimensional (2D) nonstationary wavelet filter banks based on the bivariate Bézout equation in $\mathbb{R}[z_1, z_2]$, extending the approach introduced in [1]. Since the Fejér-Riesz factorization does not extend to two variables, a tensor-product sum-of-squares (TP-SOS) decomposition is used to ensure positivity, leading to a semidefinite programming (SDP) formulation. Several design criteria, including minimal support, vanishing moments, and sparsity, are considered. Numerical results show improved performance compared to separable and classical tensor-product wavelet filters.

Keywords: 2D nonstationary wavelets, wavelet filter banks, sum-of-squares (SOS).

References:

[1] M. Brahimi, K. Melkemi and A. Boussaad, Design of nonstationary wavelets through the positive solution of Bézout's equation. J. Interdiscip. Math. 24 (2021), no. 3, 553–565.

[2] I. Daubechies, Ten Lectures on Wavelets. CBMS-NSF Regional Conf. Ser. in Appl. Math., Vol. 61, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1992.

[3] J. B. Lasserre, Global optimization with polynomials and the problem of moments. SIAM J. Optim. 11 (2000/01), no. 3, 796–817.

[4] P. A. Parrilo, Semidefinite programming relaxations for semialgebraic problems. Math. Program. 96 (2003), no. 2, Ser. B, 293–320.

[5] C. Vonesch, T. Blu and M. Unser, Generalized Daubechies wavelet families. IEEE Trans. Signal Process. 55 (2007), no. 9, 4415–4429.