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On the Isospectrality Relation Between PT-Symmetric Schrödinger Operators and Their Adjoints

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Abstract

We consider the one-dimensional Schrödinger operator $L(q)y = -y'' + q(x)y$ generated by a complex-valued, locally integrable, 1-periodic potential q satisfying the PT-symmetry condition $\overline{q(-x)} = q(x)$. Such operators are, in general, non-self-adjoint, yet they form a distinguished class whose spectra retain several features of the self-adjoint setting. This talk concerns a structural property underlying that behaviour: the adjoint $L^*(q)$ of a PT-symmetric operator $L(q)$ is again PT-symmetric. We then indicate how this property leads to the spectral equality $\sigma(L(q)) = \sigma(L^*(q))$ between the operator and its adjoint, so that the reality of the spectrum in this class can be traced back to the isospectrality between $L(q)$ and $L^*(q)$ rather than to self-adjointness.

Keywords: PT-symmetry, non-self-adjoint Schrödinger operator, adjoint operator, isospectrality.

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