



Approximation in Weighted Spaces by Bivariate Max-Product Operators

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Abstract

In this paper, we introduce a new weighted setting for the study of approximation properties of bivariate max-product operators. A weighted modulus of continuity adapted to the underlying approximation process is constructed, and its fundamental properties are established. This approach is used to examine the approximation properties of bivariate max-product Favard–Szász–Mirakyan and Meyer–König–Zeller operators. Quantitative estimates for the degree of approximation are obtained in terms of the proposed weighted modulus. The developed approach enables the analysis of approximation processes on both bounded and unbounded domains within a unified framework. The obtained results extend existing approximation estimates for bivariate nonlinear max-product operators and contribute to the theory of multivariate nonlinear approximation.

Keywords: Bivariate max-product operators, weighted modulus of continuity, degree of approximation.

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