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Orthogonally Additive Polynomials: A New Frontier in Riesz Space Theory

Elmiloud Chil

[(1)] *University of Tunis, Institut préparatoire aux études d'ingenieurs de Tunis 2 Rue Jawaher Lel Nehrou Monflery 1008, Tunisia*
e-mail: elmiloud.chil@ipeit.rnu.tn

Abstract

In this lecture, we focus on the systematic study of orthogonally additive polynomials as a new and promising area within Riesz space theory. A central challenge in this domain is understanding how these polynomials interact with other mathematical structures, particularly Riesz spaces. Orthogonally additive polynomials are functions that exhibit a specific form of additivity, allowing the construction of new functions through a defined additive process. This property renders them particularly useful in modeling complex systems characterized by the interaction of multiple variables.

Despite their potential, the structural properties of orthogonally additive polynomials especially within the context of Riesz spaces have remained elusive until recently. Their study is significant not only from an algebraic standpoint but also within the framework of infinite-dimensional analysis, notably in the theory of holomorphic functions on infinite-dimensional spaces.

To the best of our knowledge, Sundaresan was the first to investigate orthogonally additive polynomials, obtaining a representation theorem for polynomials on on ℓ^p and on L^p. spaces. In recent years, there has been a resurgence of interest in this class of mappings. Notable developments include research on orthogonally additive polynomials, holomorphic functions, and orthosymmetric multilinear mappings on various structures, including Banach lattices, $\mathbb{C}^*$-algebras, uniformly complete vector lattices, and their variants. These have also been studied when the range space is a separated convex bornological space or a Hausdorff topological vector space, and more recently, in the context of general vector lattices.

It is important to note that most existing results rely heavily on the representation of these spaces as vector spaces of extended continuous functions. Consequently, such methods are not applicable in more general vector lattice settings, underscoring the need for new theoretical approaches.

The innovation of the present work lies in establishing a connection between orthogonally additive polynomials and orthosymmetric multilinear mappings acting on vector lattices with values in Hausdorff topological vector spaces. This connection enables constructive proofs of results previously obtained using more restrictive assumptions. As an application, we provide a comprehensive description of the theory of such operators via a lattice representation theorem. Furthermore, we develop a Nakano-type theorem for these operators, offering a robust framework for analyzing their behavior within this broader context.