



Approximation by Deferred Means of Hexagonal Fourier Series

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Abstract

In this study, the convergence and approximation properties of Fourier series defined on two-dimensional hexagonal domains are investigated. The rate of approximation results obtained in the literature for classical de la Vallée Poussin means via linear positive operators are generalized in this work to "Deferred de la Vallée Poussin" means, which offer a more flexible structure based on parameter sequences. To overcome the analytical difficulties on the hexagonal lattice, the deferred means of hexagonal Fourier series are constructed by utilizing the intersection geometry and the modulo 3 congruence relation provided by the three-dimensional homogeneous coordinate system. Finally, the rates of approximation (error bounds) of these newly defined operators to continuous functions in the Lipschitz class are proved in terms of the modulus of continuity.

Keywords: Hexagonal domain, Fourier series, rate of approximation, deferred de la Vallée Poussin means.

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