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Fluctuationlessness Theorem Applied to Caputo Fractional Initial Value Problems

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Abstract

We adapt the Fluctuationlessness Theorem [1]–which represents a scalar function of the independent-variable operator, on a finite-dimensional subspace, by that function evaluated on the matrix of the independent variable once fluctuation terms are neglected–to the numerical solution of Caputo fractional initial value problems. Rewriting such a problem as a weakly singular Volterra integral equation, we discretize the memory integral panel by panel: on each step the integral splits into regular history panels and a single weakly singular diagonal panel. The diagonal panel collapses exactly, and independently of the mesh, onto a normalized Jacobi weight $w_\alpha(\sigma) = \alpha(1-\sigma)^{\alpha-1}$, for which the theorem supplies a quadrature–computed once and stored–whose nodes accumulate at the endpoint singularity. Predictor-corrector marching schemes are built on this splitting [2]. Because the diagonal panel is treated exactly and mesh-independently, the method accommodates graded meshes that cluster points near the initial singularity, avoiding the accuracy loss suffered by closed-form product weights [3]. A detailed error and convergence analysis on graded meshes is developed in a separate, fuller study. Linear, nonlinear, and weakly singular numerical tests illustrate the accuracy of the approach.

Keywords: Fluctuationlessness theorem, Caputo fractional derivative, product integration, weakly singular Volterra equation, Jacobi weight, graded mesh, predictor-corrector.

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