k-Hypergeometric Functions and Applications

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Abstract

Hypergeometric functions play a central role in the theory of special functions, as they provide a common framework for studying the solutions of many differential equations [3]. With suitable choices of parameters, they reduce to several well-known functions such as Bessel and Legendre functions. A natural extension of these classical structures is given by the k-hypergeometric functions, which are defined in terms of the k-Gamma function and the k-Pochhammer symbol, and were introduced by Diaz and Pariguan [1].

In this work, we first recall some basic properties of classical hypergeometric series and then examine how these structures can be generalized by means of the k-parameter. The main focus is on the k-extensions of balanced and well-balanced hypergeometric series. In this context, k-analogues of several classical identities and summation formulas—such as those of Saalschütz, Watson, Dixon, Whipple, and Bailey—are also considered. In particular, their convergence conditions and certain fundamental properties are discussed.

The results indicate that these k-generalized structures can be regarded as a natural continuation of classical hypergeometric functions and that they recover several known identities in the literature. From this perspective, the study highlights the role of parametric generalizations in the development of hypergeometric function theory [1, 4].

Keywords: Hypergeometric identities, generalized k-hypergeometric functions, k-pochhammer symbol.

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