



Existence of Solutions for Fractional Differential Equation with Nonlocal Conditions

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Abstract

In this paper, we consider the following fractional differential equation

$$D^\alpha u(t) + f(t, u) = 0, \quad t \in (0, 1), 1 < \alpha \leq 2, \quad (1)$$

with nonlocal boundary conditions,

$$u(0) = \int_0^1 g_1(u(t)) dt, \quad u(1) = \int_0^1 g_2(u(t)) dt, \quad (2)$$

where D^α is the Caputo fractional derivative of order $1 < \alpha \leq 2$, $f : [0, 1] \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous, and $g_1, g_2 : \mathbb{R} \rightarrow \mathbb{R}$ are continuous.

By applying lower and upper solutions method and the Schauder fixed theorem, we obtain the existence of least one solution for problem (1) – (2).

Keywords: Fractional differential equations, nonlocal conditions, upper and lower solutions, fixed point.

References:

- [1] R. P. Agarwal, M. Benchohra and S. Hamani, Boundary value problems for differential inclusions with fractional order. *Adv. Stud. Contemp. Math. (Kyungshang)* 16 (2008), no. 2, 181–196.
- [2] A. Boucherif and S. K. Ntouyas, Nonlocal initial value problems for first order fractional differential equations. *Dynam. Systems Appl.* 20 (2011), no. 2-3, 247–259.
- [3] A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*. Elsevier, Amsterdam, 2006.