



A Unified h - γ Blossoming and Bézier Curves for Translation Invariant Spaces

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Abstract

Blossoming is a fundamental tool in geometric design, underlying many constructions and algorithms for Bézier and related curve representations. In this work, we introduce a novel h - γ blossom for translation invariant (γ_1, γ_2) spaces generated by two continuous, linearly independent functions. This class includes polynomial, trigonometric, and hyperbolic function spaces and their discrete analogues. The proposed h - γ blossom merges γ -blossoming with h -blossoming within a unified framework for studying h - γ Bernstein basis functions and Bézier curves. Within this framework, we define the corresponding h - γ Bernstein basis functions and h - γ Bézier curves, and derive recursive evaluation algorithms, subdivision procedures, degree elevation formulas, Marsden identities, and interpolation formulas. In particular, the parameter h serves not only as a new shape parameter for translation invariant (γ_1, γ_2) spaces but also gives rise to interpolation formulas that are absent from the standard γ -based setting.

Keywords: Blossoming, Bernstein bases, Bézier curves, Marsden's identity, subdivision.