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Smoothed Rank-Based Regression Estimation Using Wilcoxon Score Functions

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Abstract

This article proposes an improved rank-based regression estimator obtained by replacing the ordinary integer ranks in the Wilcoxon rank-score regression procedure with *smoothed ranks* derived from a smoothed empirical cumulative distribution function (ecdf). The smoothed ranks are computed via a continuous, non-decreasing kernel distribution function $H(\cdot)$ that provides a differentiable approximation to the classical indicator function used in standard rank regression. Substituting these smoothed ranks into the Wilcoxon score function yields a new estimator, denoted $\widehat{\beta}_{sr}$, for the slope parameter(s) of the simple and multiple linear regression model. We show that the proposed estimator inherits the robustness properties of classical rank regression while providing improved efficiency under heavy-tailed error distributions and better handling of tied observations. A Wald-type hypothesis test for the regression coefficients is derived and its asymptotic normality is established. A Monte Carlo simulation study compares $\widehat{\beta}_{sr}$ with the ordinary least-squares (OLS) estimator, the classical Wilcoxon rank regression estimator, and the Theil-Sen estimator under several error distributions including the normal, Laplace, Cauchy, and contaminated normal. The proposed estimator achieves relative efficiencies at or above those of classical rank regression uniformly across all scenarios considered, with notable gains in the presence of outliers and heavy-tailed errors.

Keywords: Rank Regression, smoothed ranks, Wilcoxon score function, robust estimation, relative efficiency, Monte Carlo simulation.

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