A Maximal Ergodic Lemma for q-Weighted Ergodic Averages

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Abstract

Let $(X, \mathcal{B}, \mu)$ be a probability space and let $T : X \to X$ be a measure-preserving transformation. For $f \in L^1(\mu)$, the classical ergodic averages are defined by

$$M_N f(x) := \frac{1}{N} \sum_{n=0}^{N-1} f(T^n x).$$

The Birkhoff pointwise ergodic theorem asserts that these averages converge almost everywhere to $\mathbb{E}(f \mid \mathcal{I})(x)$, where $\mathcal{I}$ denotes the T-invariant σ-algebra. A classical proof of Birkhoff's pointwise ergodic theorem is based on the maximal ergodic lemma. Several proofs of this lemma have appeared in the literature, including those of Garsia and Riesz. In parallel with these developments, various weighted versions of the maximal ergodic lemma have also been established.

In this talk, we introduce q-weighted ergodic averages and establish a q-weighted version of the classical maximal ergodic lemma for $0 < q \leq 1$ by adapting Garsia's method to the q-setting. Our proof follows the main ideas of the classical argument while making use of the structure of the associated q-weighted sums. This work provides a first step toward the study of q-weighted ergodic theory.

Keywords: Maximal ergodic lemma, ergodic averages, q-calculus.

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