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Improvement of Berezin Radius Inequalities on Reproducing Kernel Hilbert Spaces by Applying Cartesian Comparability

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Abstract

Utilizing the Cartesian decomposition $T = T_1 + iT_2$, we define a new family of modified Berezin radius inequalities for restricted operators on reproducing kernel Hilbert spaces. Our approach depends on a comparability condition between the positive operators $|T|$ and $|T^*|$, in opposition to the conventional method based on the pair $|T_1|$ and $|T_2|$. This clearly refines the standard Cartesian-decomposition estimate for $\mathrm{ber}^{\eta}(T)$ in the range $1 \leq \eta \leq 2$. Also, we present an explicit endpoint formula for the improvement constant, establish a convex-function version of the main result, and also show that this constant is the best. The sharpness and gain of the new boundaries are shown by a number of instances on a discrete reproducing kernel Hilbert space.

Keywords: Berezin number, reproducing kernel Hilbert space, Cartesian decomposition, operator inequalities, convexity.

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