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Non-Isotropic Approximation by Multidimensional Mellin-Type Singular Integral Operators: Asymptotic and Quantitative Results

Harun Karsli

Bolu Abant Izzet Baysal University, Department of Mathematics, Bolu, Türkiye
e-mail: karsli_h@ibu.edu.tr

Abstract

We introduce and investigate a family of multidimensional Mellin-type singular integral operators acting on the multiplicative group $\mathbb{R}_+^n = (0,\infty)^n$ endowed with a λ-non-isotropic metric structure. The proposed framework combines the multiplicative nature of Mellin analysis with a non-isotropic geometry that allows different scaling behaviors along different coordinate directions.

Under suitable normalization, localization, and logarithmic moment assumptions on the kernel family, we establish some convergence results of the operators and derive quantitative estimates in terms of the λ-non-isotropic distance. Furthermore, by means of a multidimensional Mellin–Taylor expansion, we derive an asymptotic expansion of the approximation process and prove a Voronovskaya-type theorem expressed through Mellin differential operators.

The obtained results provide a unified framework encompassing approximation, asymptotic behavior, and Voronovskaya-type results for multidimensional Mellin-type singular integral operators in non-isotropic settings.

Keywords: Mellin-type singular integral operators, non-isotropic approximation, asymptotic expansion, Voronovskaya theorem, quantitative approximation, multiplicative harmonic analysis.

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