Approximation Theory and Special Functions
ATSF 2026 Conference - 9th Series
September 2-5, 2026, Ankara - Türkiye

Delay-Induced Instability and Chaos in a Plankton-Fish Model Regulated by Toxins, Fear and Refuge

Hasen Mekki Öztürk

Ordu University, Department of Mathematics, Ordu, Türkiye
e-mail: hasenozturk@gmail.com

Abstract

Temporal lags in maturation are a recognised source of instability in aquatic food webs [3], yet their effect remains unexplored in a food chain already regulated by toxin-mediated mortality [1], predator fear [5] and a density-dependent refuge [2]. We extend a recent ordinary differential equation model of three species [4] comprising phytoplankton, zooplankton and fish by introducing a single discrete maturation delay into the fish recruitment term, representing the time between predation on zooplankton and its contribution to the adult fish stock. Using this delay as the primary bifurcation parameter, we analytically establish the well-posedness, positivity and uniform boundedness of solutions. The coexistence equilibrium, locally stable in the underlying ordinary differential equation system, loses stability through a supercritical Hopf bifurcation once the delay crosses a critical threshold. Numerical continuation beyond this threshold traces a cascade of period-doubling and fold bifurcations, and a positive maximal Lyapunov exponent together with a folded Poincaré section are consistent with a transition to chaotic dynamics at larger delays. These results illustrate how a biologically motivated maturation delay is sufficient to disrupt trophic stability, driving an otherwise well-regulated ecosystem from stable coexistence into dynamics consistent with chaos.

Keywords: Plankton-fish dynamics, stability analysis, maturation delay, bifurcation analysis, chaotic dynamics.

References:

[1] J. Chattopadhyay, R. R. Sarkar, and S. Mandal, Toxin-producing plankton may act as a biological control for planktonic blooms-field study and mathematical modelling. J. Theoret. Biol. 215 (2002), no. 3, 333–344.

[2] E. González-Olivares and R. Ramos-Jiliberto, Dynamic consequences of prey refuges in a simple model system: more prey, fewer predators and enhanced stability. Ecological Modelling 166 (2003), no. 1-2, 135–146.

[3] Y. Kuang, Delay Differential Equations with Applications in Population Dynamics. Academic Press, Boston, 1993.

[4] A. Mandal, N. Sk and S. Biswas, Nutrient enrichment and phytoplankton toxicity influence a diversity of complex dynamics in a fear-induced plankton-fish model. J. Theoret. Biol. 578 (2024), Paper No. 111698, 12 pp.

[5] X. Wang, L. Zanette and X. Zou, Modelling the fear effect in predator-prey interactions. J. Math. Biol. 73 (2016), no. 5, 1179–1204.