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Kelvin Inversion Polynomials: Orthogonality and Structure

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Abstract

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The Kelvin transform is a classical tool in potential theory that maps harmonic functions to harmonic functions. In this talk, we consider a family of multivariate polynomials naturally arising from the Kelvin transform and its infinitesimal generators. These polynomials, which we call *Kelvin inversion polynomials*, are defined recursively by

$$p_0 = 1, \qquad p_{\alpha+\epsilon_j} = \mathcal{D}_j p_\alpha,$$

where $\mathcal{D}_j = |\mathbf{x}|^2 \partial_{x_j} - x_j(n + 2\mathbb{E})$ and $\mathbb{E} = \sum_{k=1}^n x_k \partial_{x_k}$ is the Euler operator. Equivalently, they appear as the numerators in the derivatives of the fundamental solution of Laplace's equation:

$$\partial_x^\alpha \frac{1}{|\mathbf{x}|^n} = \frac{p_\alpha(\mathbf{x})}{|\mathbf{x}|^{n+2|\alpha|}}.$$

Such polynomials arise naturally when studying solutions to the Dirac equation associated with Clifford-Kanzaki algebras (see [2]). In this talk, we examine an operator algebra acting on polynomials and use it to analyze their orthogonality.

Keywords: Kelvin transform, orthogonal polynomials, conformal algebra, Casimir operator, potential theory, harmonic functions.

References:

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