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On the Approximation of Borel Derivatives by Nonlinear Durrmeyer Type Operators

Hüseyin Erhan Altın[(1)] and Harun Karslı[(2)]

[(1)] *Bolu Abant İzzet Baysal University, Department of Mathematics, Bolu, Türkiye*
e-mail: erhanaltin@ibu.edu.tr

[(2)] *Bolu Abant İzzet Baysal University, Department of Mathematics, Bolu, Türkiye*
e-mail: karsli_h@ibu.edu.tr

Abstract

In the present study, we consider a sequence of nonlinear Durrmeyer type operators (ND_n) of the form

$$(ND_n)(f;x) = \int_0^1 K_n(x,t,f(t))\,dt, \quad x \in [0,1], \quad n \in \mathbb{N},$$

acting on Lebesgue measurable functions defined on $[0,1]$ where $K_n(x,t,u)$ satisfy some suitable assumptions.

We give some approximation results with regard to the convergence of the operators ND_n to right, left, and symmetric Borel differentiable functions.

Our study extend some of the previous results about the linear integral operators and also aims to bring a new perspective to research in approximation with nonlinear operators by introducing a different concept of a derivative.

Keywords: Borel derivative, nonlinear operator, pointwise convergence.

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