Determination of a Radially Symmetric Potential in anIinverse Scattering Problem

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Abstract

This talk concerns an inverse problem in scattering of plane waves by a medium. The forward problem consists to find the far field pattern (or scattering amplitude) from the scatterer (inhomogeneity in the medium). The total wave $u = u^s + u^i$ satisfies the Helmholtz equation

$$\triangle u + k^2 p(x) u = 0 \quad \text{in} \quad \mathbb{R}^2$$

with the radiation condition

$$\lim_{r \to +\infty} \sqrt{r} \left(\frac{\partial u^s}{\partial r} - iku^s \right) = 0.$$

For incident plane wave $u^i(x) = \exp(ikd \cdot x)$, the far-field $u^\infty(\hat{x})$ is defined by the asymptotic formula

$$u^s(x) = \frac{e^{ikr}}{\sqrt{r}} \left\{ u^\infty(\hat{x}) + \mathcal{O}\left(\frac{1}{r}\right) \right\}, \quad r = |x|, \quad \hat{x} = \frac{x}{r} = (\cos\theta, \sin\theta).$$

More precisely we have ([2])

$$u^\infty(\hat{x}) = e^{i\frac{\pi}{4}} \sqrt{\frac{k^3}{8\pi}} \int_{\mathbb{R}^2} e^{ik\hat{x}\cdot y} q(y) u(y) dy \quad \left(q = 1 - p \in C_0^2(\mathbb{R}^2)\right).$$

We assume that $p(x) = p(r)$ such that $p(r) = 1$ for $r > R$. The aim of this work is to derive the forward operator $F : q \mapsto u^\infty(\theta) = \sum_{n=0}^{\infty} S_n(q) \cos n\theta$. Then we study the inverse problem (determining the potential q from the far field) which is equivalent to the nonlinear equation $F(q) = u^\infty_{ob}$ where u^∞_{ob} is the observed far field.

Keywords: Inverse scattering, radial symmetry, far field data.

References:

[1] J. Shin and E. Arhin, Determining radially symmetric potential from near-field scattering data. J. Appl. Math. Comput. 62 (2020), no. 1-2, 511–524.

[2] F. Cakoni and D. Colton, A Qualitative Approach to Inverse Scattering Theory. Appl. Math. Sci., Vol. 188, Springer, New York, 2014.