Hardy-Rogers Type Fixed Point Results for Quantale-Valued Quasi-Metric Spaces

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Abstract

Metric spaces have been generalized by modifying the classical metric axioms. For example, by removing the symmetry condition, quasi-metric spaces were described in [3]. Another important generalization of metric spaces is partial metrics and these structures were introduced by Matthews [6]. Partial metric spaces play an important role in computer science. In 1977, Flagg [2] defined the quantale-valued quasi metric spaces by changing the set $[0, \infty]$ with a value-quantale. In [1], the notion of an action was defined and Banach contraction principle was extended to the setting of quantale-valued quasi-metric spaces. These structures are generalization of both metric spaces and probabilistic metric spaces [8], and they are important tools in quantitative domain theory and denotational semantics. In [4], Hardy and Rogers obtained an important generalization of Reich's fixed point result given in [7]. In this work, we first present some important topological properties of quantale-valued quasi metric spaces. Then, we extend the Hardy-Rogers type fixed point results to the setting of quantale-valued quasi-metric spaces and we derive some consequences in both probabilistic and partial quantale-valued metric spaces [5]. Our results generalize the results given in the literature.

Keywords: Value-quantale, fixed point, complete lattice, action.

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