Numerical Solution of Nonlinear Volterra Integro-Differential Equations via Spline Approximation

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Abstract

In this work, we consider a system of nonlinear Volterra integro-differential equations. We propose a spline collocation method for the numerical solution of such problems. The method is formulated in suitable spline approximation space, where Lagrange interpolation at the collocation points is employed to approximate the unknown solution and its derivative.

Explicit representations of the approximate solution are derived, leading to a direct computational procedure that avoids the solution of large algebraic systems and the evaluation of complicated integrals. A rigorous convergence analysis is established, demonstrating that the method achieves a high order of accuracy.

Several numerical examples are presented to validate the theoretical results. The computational experiments confirm the convergence and accuracy of the proposed method. Comparisons with well-known existing methods illustrate the efficiency and reliability of the present approach.

Keywords: Volterra integro-differential equations, collocation method, cpline polynomials, convergence analysis.

References:

[1] A. Bellour, M. Bousselsal and H. Laib, Numerical solution of second-order linear delay differential and integro-differential equations by using Taylor collocation method. Int. J. Comput. Methods 17 (2020), no. 9, 1950070, 31 pp.

[2] H. Brunner, Collocation Methods for Volterra Integral and Related Functional Equations. Cambridge University Press, Cambridge, 2004.

[3] H. Laib, A. Bellour and A. Boulmerka, Taylor collocation method for a system of nonlinear Volterra delay integro-differential equations with application to COVID-19 epidemic. Int. J. Comput. Math. 99 (2022), no. 4, 852–876.

[4] H. Li and J. Ma, On generalized multistep collocation methods for Volterra integro-differential equations. Math. Comput. Simulation 226 (2024), 399–412.

[5] W. Zheng, Y. Chen and J. Zhou, A Legendre spectral method for multidimensional partial Volterra integro-differential equations. J. Comput. Appl. Math. 436 (2024), Paper No. 115302, 17 pp.