A Generalization of Banach's Closed Range Theorem

Krzysztof Zakrzewski

Warsaw School of Economics, Collegium of Economic Analysis, Warsaw, Poland
e-mail: kzakrze@sgh.waw.pl

Abstract

For a real topological vector space X, let X^* denote its dual space, that is the space of linear continuous functionals endowed with the $weak^*$ topology. For a continuous linear operator $T : X \to Y$ between two topological vector spaces, by $T^* : Y^* \to X^*$ we denote the dual operator of T.

A classical theorem of Banach states that a continuous linear operator $T : X \to Y$ between two Banach spaces has closed range if and only if its dual operator $T^* : Y^* \to X^*$ has closed range. A generalization of this theorem to the setup of general locally convex topological vector spaces can be found in [1] [Theorem 7.3/Chapter IV]. This generalization states that a continuous linear operator $T : X \to Y$ between two locally convex topological vector spaces is $weak - weak$ open onto its image if and only if T^* has closed range. The main theorem proved during this talk is a symmetric generalization of Banach's closed range theorem: a continuous linear operator $T : X \to Y$ between two locally convex topological vector spaces has closed range if and only if T^* is $weak^* - weak^*$ open onto its image.

Keywords: Dual operator, closed range theorem, topological vector spaces.

References:

[1] H. H. Schaefer, Topological Vector Spaces. Springer Verlag, New York, 1971.