A Kernel-Based Infeasible Primal-Dual Interior Point Method for Convex Nonlinear Optimization

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Abstract

Interior Point Methods (IPMs) are among the most powerful frameworks for solving large-scale convex optimization problems [1]. In their classical *feasible* version, however, they require a strictly feasible starting point, a condition often costly to ensure. *Infeasible* variants (IIPMs) [4] overcome this limitation by allowing an arbitrary starting point in the positive orthant, while simultaneously reducing primal infeasibility, dual infeasibility, and the duality gap along a perturbed central path.

In this talk, we present a family of primal–dual IIPMs designed for **convex nonlinear problems**, in which the classical logarithmic barrier is replaced by a **kernel function** ψ satisfying the eligibility conditions K_1–K_4 introduced by Peng, Roos and Terlaky [2]. The associated proximity function

$$\Psi(v) = \sum_{i=1}^{n} \psi(v_i), \qquad v_i = \sqrt{\frac{\lambda_i s_i}{\mu}},$$

serves both as a centrality measure and as a merit function driving the line search.

We will discuss: (i) the design of the kernel function and its influence on the geometry of the central-path neighborhood [3]; (ii) the Newton step analysis in the nonlinear setting; (iii) the iteration complexity bounds, namely $\mathcal{O}(\sqrt{n}\,\log(n/\varepsilon))$ in the small-update case and $\mathcal{O}(\sqrt{n}\,(\log n)\,\log(n/\varepsilon))$ in the large-update case; and (iv) numerical experiments highlighting the robustness of the approach and the practical impact of the kernel choice on convergence behaviour [5].

Keywords: Convex nonlinear optimization, kernel function, infeasible primal-dual interior point method, proximity function, polynomial complexity.

References:

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