On the Space of Sequential Modulus Approximately Continuous Function

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Abstract

In this paper, we study the space of sequentially γ-modulus approximately continuous functions with respect to a fixed sequence $\bar{t} = \{t_n\} \in \mathbb{S}$. We first show that the class $\mathcal{SA}_{\gamma,\bar{t}}$ of all sequentially γ-modulus approximately continuous real-valued functions forms a vector space under pointwise addition and scalar multiplication. Then, we consider the bounded subspace $\mathcal{SA}^{\infty}_{\gamma,\bar{t}}$ and prove that it becomes a normed linear space under the supremum norm: $\|f\|_\infty = \sup_{x\in\mathbb{R}} |f(x)|$ Moreover, we show that this space is complete, and hence it is a Banach space. We also compare this space with the classical spaces of continuous functions and bounded continuous functions. In particular, we prove that: $C(\mathbb{R}) \subset \mathcal{SA}_{\gamma,\bar{t}}$ and $C_b(\mathbb{R}) \subset \mathcal{SA}^{\infty}_{\gamma,\bar{t}}$ but show that these inclusions are proper. This demonstrates that sequential γ-modulus approximate continuity provides a broader framework than classical continuity, allowing functions to exhibit sparse exceptional oscillations while preserving a rigorous density-type limiting behavior.

Keywords: Lebesgue measure, density point, Lebesgue density theorem, modulus density point.

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