Numerical Floquet Theory: The Dynamics of Numerical Methods

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Abstract

Nonautonomous differential equations possessing periodic solutions arise naturally in a wide range of scientific and engineering applications, including mechanics, electrical circuits, control theory, and biological models subject to periodic forcing. Of particular interest are systems for which the period of the exact solution is known a priori, as this structural information can be exploited in both analysis and numerical approximation.

In this presentation, we focus on a class of simple nonautonomous systems whose solutions are periodic with a prescribed period. We investigate how such systems are transformed when discretized by linearized collocation methods. To analyze the qualitative behavior of the resulting numerical solutions, we employ stroboscopic sampling, which can be viewed as a discrete analogue of the classical Poincaré map obtained by sampling the continuous solution once per period. This approach reduces the study of periodic orbits to the analysis of fixed points of an associated discrete dynamical system.

Using this framework, we examine the stability properties of the numerical schemes and their ability to reproduce the correct long-term behavior of the underlying continuous system. In particular, we demonstrate that there is an inextricable connection between AN-stability (or BN-stability) of the collocation methods and the faithful qualitative reproduction of periodic solutions. Methods lacking the appropriate stability properties may generate spurious growth or decay in the stroboscopic map, leading to incorrect conclusions about the stability of the true periodic orbit. Our results highlight the crucial role of numerical stability theory in the reliable simulation of nonautonomous periodic systems and provide guidance for the selection of collocation schemes in practical computations.

Keywords: Floquet theory, dynamics of numerics, AN- and BN-stability.

References:

[1] C. Pötzsche, D. Rackl, Nyström discretization of integrodifference equations: numerical continuation of periodic solutions and Floquet multipliers. Numer. Algorithms 98 (2025), no. 3, 1429–1465.

[2] D. J. Higham, Runge-Kutta stability on a Floquet problem. BIT 34 (1994), no. 1, 88–98.

[3] P. Deuflhard and F. Bornemann, Scientific Computing with Ordinary Differential Equations. Springer-Verlag, New York, 2002.