



Approximation of Entire Functions in Some Banach Spaces

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Abstract

This paper delves into the intricate relationship between the generalized order of an entire function, defined by a model function, and the rate at which this function can be approximated by polynomials within the unit disk. Our investigation unveils novel connections, establishing a precise relationship between the generalized order of the entire function and the convergence of its best polynomial approximations.

In this article, we examine entire functions whose growth is governed by the model function proposed by B. N. Khabibullin in [1] (see also [2]). This study explores the relationship between the growth of entire functions and their inherent properties and is a continuation of the research of V.B. Vakarchuk and his followers (see, for example [4]).

Definition. Let $\Psi(r)$ is the model function of growth. Then

$$\limsup_{r \rightarrow \infty} \frac{\log \log \mathcal{M}(r, f)}{\log \Psi(r)} = \varrho_{\Psi} \in \mathbb{R}_+ \quad (1)$$

are called the order of the function f relatively to the model function of growth Ψ .

Theorem. Let ϱ_{Ψ} be the order of growth of the entire function, defined by (1) where Ψ is the function of growth which satisfies the some conditions (see [3]). Then the entire function $f \in B_{p,q,\lambda}$ has generalized order ϱ_{Ψ} if and only if

$$\varrho_{\Psi} = \limsup_{k \rightarrow \infty} \frac{\log k}{\log(\Psi((E_n(f, B_{p,q,\lambda}))^{-k^{-1}}))}.$$

Keywords: Entire functions, model function, growth function, generalized order, polynomial approximation.

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