



Fekete-Szego Inequalities and Hankel Determinant Estimates for a Novel Bi-Univalent Functions Associated with Horadam Polynomials

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Abstract

In Geometric function theory of complex analysis, analytic and bi-univalent functions play an important role in applied analysis, particularly in problems involving conformal mappings, fluid flow modeling, elasticity theory, and other areas where geometric transformations are fundamental. The development of defining new subclasses of univalent and bi-univalent functions provides a useful framework for generating models with controlled geometric behavior. However, there is a necessity to generalize certain subclasses of analytic functions to widen the range of their applications.

Hence, a new subclass of analytic and bi-univalent functions $\mathfrak{R}_{\Sigma}(n, \lambda, \gamma, \mu, \beta, t, \phi)$ is introduced and defined in the unit $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ by using Opoola differential operator. The proposed subclass of bi-univalent functions $\mathfrak{R}_{\Sigma}(n, \lambda, \gamma, \mu, \beta, t, \phi)$ offers a flexible structure that unifies and extends several existing operators and subclasses in geometric function theory.

Using the principle of subordination together with properties of Horadam polynomial, the upper estimates for the initial-Taylor Maclaurin coefficients was explicitly derived. In addition, upper bounds of Fekete-Szego functional $|a_3 - \xi a_2^2|$ and Hankel determinant $H_2(f(z))$ which are important quantities in stability analysis and geometric modeling are obtained. The results not only generalize several known results in the literature but also provide analytical tools that may be useful in areas where conformal structures and analytic transformations play an important role.

Keywords: Analytic and bi-univalent functions, Opoola differential operator, Fekete-Szego functional, Hankel determinant, subordination principle.

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