



## Spline-Based Laplace-Stieltjes Approximation for Saturated Generalized-Gamma Aggregates

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### Abstract

Generalized-Gamma (GG) distributions provide a flexible framework for modeling fading, reliability, and aggregate uncertainty in engineering systems. In many practical settings, however, the effective contribution of each component is subject to saturation or clipping, leading to aggregate variables of the form.

$$S_N = \sum_{i=1}^N a_i \min(G_i, T_i),$$

where  $G_i$  follows a GG distribution,  $a_i > 0$  is a deterministic weight, and  $T_i$  is a saturation threshold. Unlike ordinary sums of GG variables, each saturated component has a mixed distribution, with a continuous part on  $[0, T_i)$  and a point mass at  $T_i$ , which makes the distributional analysis of SN nontrivial.

This work proposes a cubic-spline-assisted Laplace–Stieltjes transform approximation for the distribution of such saturated aggregates. The point mass induced by saturation is treated exactly in the transform domain, while cubic splines approximate the continuous GG density over the unsaturated region. This leads to finite polynomial-exponential integral expressions for the component transforms. By independence, the transform of  $S_N$  is obtained as a product of the weighted component transforms, enabling efficient evaluation of the probability density function and cumulative distribution function, and outage-type performance metrics. The proposed approach is applied to outage analysis of clipped diversity receivers over GG fading channels. Numerical comparisons with Monte Carlo simulations show close agreement across representative fading and saturation regimes, confirming that the spline-assisted Laplace–Stieltjes approximation provides a stable and accurate tool for performance analysis of saturated stochastic systems.

**Keywords:** Generalized-Gamma distribution, cubic spline approximation, Laplace–Stieltjes transform, saturated random variables, outage probability, wireless fading.

### References:

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