



Some Spectral and Numerical Radii Bounds

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Abstract

In this work, we first discuss in further detail new possible and practical bounds for the spectral radius of Hilbert space operators. These bounds are related to the celebrated Davidson-Power type inequality, as a new direction in the discussion of spectral radii. Then, numerical radius bounds are shown in a way that complements some known bounds. The new results are compared with their counterparts from the literature, providing a comprehensive overview of these findings.

Keywords: Spectral radius, numerical radius, operator norm, Davidson-power inequality.

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