$\mathcal{L}^2$-Consistent and Asymptotically Normal Quasi-Maximum Likelihood Estimators for Milstein-Discretized Scalar Diffusion Process

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Abstract

Although the Milstein scheme achieves strong-order-1 pathwise accuracy for stochastic differential equations, the $\mathcal{L}^2$-consistency and asymptotic distribution of parameter estimators derived from it have not been rigorously established. This paper closes that gap by developing a quasi-maximum likelihood estimation (QMLE) framework for Milstein-discretized scalar diffusion, using geometric Brownian motion as a benchmark. Drift and volatility estimators are constructed via martingale estimating functions on conditional moments of the Milstein scheme, avoiding evaluation of the exact transition density. $\mathcal{L}^2$-consistency is proved under in-fill asymptotics using Gaussian moment identities and the Marcinkiewicz-Zygmund inequality; asymptotic normality is established with closed-form asymptotic variances, and joint Wald confidence regions are justified. Monte Carlo simulations show that Milstein-QMLE achieves lower absolute error than Euler-Maruyama-based MLE (Euler-MLE) on coarse grids; under high volatility, Euler-MLE fails positivity preservation in 70.7% of paths versus 0.0% for Milstein-QMLE. An empirical S&P 500 application yields a 27% reduction in out-of-sample mean-squared error.

Keywords: $\mathcal{L}^2$-Consistency, asymptotic normality, Milstein scheme, quasi-maximum likelihood, parameter estimation, empirical Analysis.

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