



Thin-Domain Flow of Electrorheological Fluids with Variable Exponent and Tresca Boundary Conditions

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Abstract

This article investigates the steady flow of an incompressible non-Newtonian fluid in a thin three-dimensional domain. The fluid viscosity is governed by a variable power-law model, while external body forces act throughout the domain. On a portion of the boundary, the wall friction is modeled by the Tresca law. The resulting mathematical formulation leads to a system of nonlinear variational inequalities. The main objective is to analyze the asymptotic behavior of the solution as the thickness of the domain tends to zero. By applying a suitable change of variables, we rigorously derive the corresponding limit problem while preserving the intrinsic nonlinearity of the flow. This asymptotic analysis leads to a generalized Reynolds-type equation describing the effective behavior of the fluid in the thin-film regime.

Keywords: Non-Newtonian fluid, Reynolds equation, Variational formulation, Tresca law.

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