



Generalized Hermite-Hadamard Inequalities via the K_P -Fractional Integral Operator

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Abstract

The Hermite-Hadamard inequality is a cornerstone of convex analysis and has attracted considerable attention due to its numerous applications in mathematical inequalities, numerical analysis, optimization, and fractional calculus. In this work, we establish new Hermite-Hadamard-type inequalities involving the generalized fractional integral operator K_P , defined by

$$K_P[f](t) = \lambda \int_a^t k(t, s)f(s)ds + \mu \int_t^b k(s, t)f(s)ds,$$

where $P = \langle a, t, b, \lambda, \mu \rangle$ is a parameter set and $k(\cdot, \cdot)$ is a suitable kernel function. This operator unifies a broad class of classical fractional integral operators through appropriate choices of the kernel and the parameters.

Under suitable generalized convexity assumptions, we derive new upper and lower Hermite-Hadamard-type inequalities associated with the operator K_P . The obtained results generalize several existing inequalities in the literature and recover many well-known fractional integral inequalities as particular cases. Furthermore, we discuss applications of the established inequalities to generalized fractional operators, numerical integration, and integral equations. The proposed framework provides a unified approach for investigating integral inequalities in fractional calculus and offers new perspectives for future developments in generalized convexity and applied analysis.

Keywords: Hermite-Hadamard inequality, generalized fractional integral operator K_P , generalized convexity, fractional calculus, integral inequalities, convex functions, numerical integration, integral equations.

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