



Szász-Durrmeyer Type Degenerate Hermite Operators

Mustafa Kara and Mehmet Ali Özarslan

*Eastern Mediterranean University, Faculty of Arts and Sciences, Department of Mathematics,
Gazimagusa, TRNC, Mersin 10, Türkiye
e-mail: mustafa.kara@emu.edu.tr
e-mail: mehmetali.ozarslan@emu.edu.tr*

Abstract

In this paper, we introduce a new class of Szász-Durrmeyer type positive linear operators constructed by combining the parametric structure of degenerate Hermite polynomials with integral kernels of Gamma type, providing a flexible generalization of several classical approximation processes. We first derive explicit formulas for the moments and central moments of the operators, which play a fundamental role in the analysis of their approximation behavior. By using Korovkin-type theorems on unbounded intervals, the uniform convergence of the operators to the target function is established in suitable weighted spaces. Furthermore, quantitative estimates for the rate of convergence are obtained by means of the classical modulus of continuity, second-order modulus of smoothness, Peetre's K -functional, and Ditzian-Totik type tools. Voronovskaya-type asymptotic results are also investigated. In addition, graphical simulations and numerical examples are presented to illustrate the convergence behavior of the operators for different parameter values. The results demonstrate that the introduced family provides an effective and natural extension of classical Szász-type operators within the framework of degenerate Hermite polynomials.

Keywords: Degenerate Hermite polynomials, Szász-Durrmeyer type operators, rate of convergence, weighted approximation.

References:

- [1] G. Krech, A note on some positive linear operators associated with the Hermite polynomials. *Carpathian J. Math.* 32 (2016), no. 1, 71–77.
- [2] M. Ayman-Mursaleen, Md. Heshamuddin, N. Rao, B. K. Sinha and A. K. Yadav, Hermite polynomials linking Szász-Durrmeyer operators. *Comput. Appl. Math.* 43 (2024), no. 4, Paper No. 223, 17 pp.
- [3] N. Raza, M. Kumar and M. Mursaleen, Approximation operators explored through the lens of degenerate Hermite polynomials. *Numer. Algorithms* 100 (2025), no. 3, 1179–1198.
- [4] N. Rao, A. K. Yadav, M. Mursaleen, B. K. Sinha and N. K. Jha, Szász-Beta operators via Hermite polynomial. *J. King Saud Univ. Sci.* 36 (2024), no. 4, Article No. 103120, 6 pp.
- [5] R. P. Gupta and G. C. Jain, A generalized Hermite distribution and its properties. *SIAM J. Appl. Math.* 27 (1974), 359–363.
- [6] P. Appell and J. Kampé de Fériet, *Fonctions Hypergéométriques et Hypersphériques: Polynômes d'Hermite*. Gauthier-Villars, Paris, 1926.
- [7] W. A. Khan, Degenerate Hermite-Bernoulli numbers and polynomials of the second kind, *Prespacetime J.* 7 (2016), no. 9, 1297–1305.