Linear Canonical Fourier-Bessel Gabor Transform and Extremal Functions

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Abstract

This talk presents a complete harmonic analysis framework for the canonical Fourier-Bessel transform (CFBT), a generalization of the classical Fourier-Bessel transform that unifies several integral transforms arising in optics and signal processing. The CFBT $\mathcal{F}_\nu^{\mathbf{m}}$ is parametrized by a matrix $\mathbf{m} \in SL(2, \mathbb{R})$ and is intrinsically linked to a quadratic deformation of the Bessel operator.

A key contribution is the factorization

$$e^{i(\nu+1)\frac{\pi}{2}\,\mathrm{sgn}(b)}\,\mathcal{F}_\nu^{\mathbf{m}} = \mathbf{L}_{\frac{d}{b}} \circ \mathbf{D}_b \circ \mathcal{F}_\nu \circ \mathbf{L}_{\frac{a}{b}},$$

where $\mathbf{L}_a$ and $\mathbf{D}_a$ are chirp multiplication and dilation operators, respectively, and $\mathcal{F}_\nu$ is the classical Fourier-Bessel transform. This factorization allows us to derive systematically the fundamental properties of the CFBT: Riemann–Lebesgue lemma, inversion formula, Plancherel theorem, Paley-Wiener theorem, and a range of uncertainty principles including Heisenberg, Hardy, Nash, Carlson, and Miyachi-type inequalities.

As a first application, we introduce the continuous canonical Bessel-Gabor transform $\mathscr{G}_g^{\mathbf{m}}$, which combines time–frequency analysis with the CFBT structure. We prove its boundedness on L^2, establish Plancherel and inversion formulae, and derive a Heisenberg-type uncertainty principle. Using reproducing kernel theory, we study the problem of approximate concentration and show that if the energy of $\mathscr{G}_g^{\mathbf{m}}f$ is concentrated in a small set, then f cannot be arbitrarily localized. Furthermore, we characterize the image space $\mathscr{G}_g^{\mathbf{m}}(L^2)$ as a reproducing kernel Hilbert space and provide explicit pointwise bounds for the kernel.

These results, published in [1], contribute a unified theory for the CFBT and demonstrate its utility in time–frequency analysis.

Keywords: Linear canonical transform, Fourier-Bessel transform, Gabor transform, uncertainty principles.

References:

[1] H. Ben Mohamed and N. Krir, Linear canonical Bessel Gabor transform. Rend. Circ. Mat. Palermo (2) 74 (2025), no. 1, Paper No. 64, 21 pp.