



Generalized Weinstein-Sobolev-Gevrey Spaces

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Abstract

We introduce the generalized Weinstein-Sobolev-Gevrey spaces $\mathcal{H}_{a,\sigma}^{s,\nu,n}(\mathbb{R}_+^{d+1})$, defined via the generalized Weinstein transform. We establish their basic properties, including completeness, continuous embeddings, and duality. Using Saitoh's theory of reproducing kernels, we solve an extremal function problem: for a bounded linear operator $\mathcal{L} : \mathcal{H}_{a,\sigma}^{s,\nu,n} \rightarrow \mathcal{H}$, we find the unique function $f_{r,h}^*$ minimizing

$$r\|f\|_{\mathcal{H}_{a,\sigma}^{s,\nu,n}}^2 + \|h - \mathcal{L}f\|_{\mathcal{H}}^2.$$

We give an explicit representation of $f_{r,h}^*$ in terms of the reproducing kernel of a modified space and provide sharp pointwise estimates. Concrete examples, including the identity and multiplier operators, are discussed in detail.

These results, published in [1], extend the theory of the CFBT to the study of extremal functions in generalized Gevrey classes, with potential applications to regularization of inverse problems.

Keywords: Sobolev-Gevrey spaces, extremal functions, reproducing kernels.

References:

- [1] H. Ben Mohamed, M. M. Chaffar and N. Krir, Extremal functions on the spaces $\mathcal{H}_{a,\sigma}^{\alpha,n}(\mathbb{R}_+^{d+1})$. Filomat 40 (2026), no. 5, 1667–1684.