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A New Generalization of Touchard-Based Appell Polynomials

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Abstract

In this study, a new generalization of the Touchard-based Appell polynomials is introduced, by the Bell and Appell polynomials. Starting from a suitably constructed generating function that builds upon the generalized Touchard polynomials, several fundamental properties are established, including a determinant representation and a recurrence relation, together with various identities obtained through the Euler derivative operator. Structural relations with the λ-Stirling numbers of the second kind are also derived, along with connections to Apostol-Bernoulli numbers of negative order and other combinatorial sequences, as well as several finite and infinite summation formulas involving the new family. In addition, 3D surface plots of certain subfamilies of this family are presented to illustrate their structural behavior.

Keywords: Touchard polynomials, Appell polynomials, determinant form, recurrence relation, Stirling numbers.

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