



On Fourier transforms of Confluent Hypergeometric Functions and Ramanujan-Type Formulas

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Abstract

We derive some Fourier transforms of confluent hypergeometric functions. We give generalizations of several well-known results involving Fourier transforms of gamma functions. In particular, the generalizations include several of Ramanujan's remarkable formulas.

Our first result is the following theorem:

Theorem. For $a, \alpha, z \in \mathbb{C}$, with $\Re(b) > 0$ and $\Re(c) > 0$,

$$\begin{aligned} & \int_{-\infty}^{+\infty} \frac{\Gamma(b+it)}{\Gamma(b+c+it)} {}_2F_2(a, c; \alpha, b+c+it; z) e^{it\xi} dt \\ &= \begin{cases} \frac{2\pi}{\Gamma(c)} e^{-b\xi} (1-e^{-\xi})^{c-1} {}_1F_1(a; \alpha; z(1-e^{-\xi})) , & \xi > 0, \\ 0, & \xi < 0. \end{cases} \end{aligned}$$

In particular, for $\alpha = c$

$$\begin{aligned} & \int_{-\infty}^{+\infty} \frac{\Gamma(b+it)}{\Gamma(b+c+it)} {}_1F_1(a; b+c+it; z) e^{it\xi} dt \\ &= \begin{cases} \frac{2\pi}{\Gamma(c)} e^{-b\xi} (1-e^{-\xi})^{c-1} {}_1F_1(a; c; z(1-e^{-\xi})) , & \xi > 0, \\ 0, & \xi < 0. \end{cases} \end{aligned}$$

Keywords: Confluent hypergeometric function, Fourier transform, Ramanujan integrals.

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