



On the Image of the Limit q -Stancu Operator

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Abstract

The limit q -Stancu operator $S_{\infty}^{q,\alpha}$, $0 < q < 1$ and $\alpha > 0$, arises as the limit form of the q -Stancu polynomials introduced by Nowak [2] and constitutes a significant generalization of the classical Bernstein operators.

Our main goal is to understand how analytic properties of functions are reflected in their images through $S_{\infty}^{q,\alpha}$. It is proved that the image of every continuous function on $[0, 1]$ can be extended analytically to a certain disk in the complex plane. It is also shown that having more derivatives at the point $x = 1$ leads to a larger domain of analyticity. In particular, if a function is infinitely differentiable at $x = 1$, then its image is an entire function. Furthermore, growth estimates for the image functions are obtained. Finally, it is established that the image of any transcendental entire function grows strictly slower than the function itself unlike the case of polynomials where $S_{\infty}^{q,\alpha}$ owns the degree preserving property.

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Keywords: q -Bernstein operator, limit q -Stancu operator, growth estimate.

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