



A Class of Generalized Gegenbauer Polynomials

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Abstract

A derivative-based extension of the classical Gegenbauer polynomials is proposed by introducing an additional independent variable, leading to a new family of bivariate polynomials. The construction is examined through its algebraic and analytic characteristics, and explicit formulas together with the corresponding generating function are established. Several identities, including recurrence formulas, differential and integral relations, are obtained, providing effective tools for investigating the proposed polynomial family. Connections with Stirling numbers and power sums are also derived, yielding a variety of combinatorial identities. Finally, the generating function is extended to a more general form, from which further identities and consequences for the new polynomial family are deduced. The results enrich the theory of generalized Gegenbauer polynomials and provide a basis for further developments in the study of special polynomial sequences.

Keywords: Gegenbauer polynomials, recurrence relation, generating function, symmetry identity.

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