



Ideal Core of Real Sequences and Linear Transformations

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Abstract

Given an ideal \mathcal{I} on the natural numbers ω and a bounded real sequence \mathbf{x} , we denote by

$$\text{core}_{\mathbf{x}}(\mathcal{I})$$

the smallest interval $[a, b]$ such that $\{n \in \omega : x_n \notin [a - \varepsilon, b + \varepsilon]\} \in \mathcal{I}$ for all $\varepsilon > 0$. Note that, if \mathcal{I} is the ideal Fin of finite subsets of ω , then

$$\text{core}_{\mathbf{x}}(\text{Fin}) = [\liminf \mathbf{x}, \limsup \mathbf{x}].$$

In addition, $\text{core}_{\mathbf{x}}(\mathcal{I}) = \{\eta\}$ if and only if $\mathcal{I}\text{-}\lim \mathbf{x} = \eta$.

Thus, given an infinite matrix $A = (a_{n,k} : n, k \in \omega)$ and ideals \mathcal{I}, \mathcal{J} on ω , we provide several relationships between the \mathcal{I} -core of \mathbf{x} , the \mathcal{J} -core of the linear transformation $A\mathbf{x}$, and certain regularity properties of A . Incidentally, we show that the unit ball of $\ell_{\infty}(\mathbf{R}^d)$ coincides with the closed convex hull of its extreme points.