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On Riemann-Liouville Type Fractional Hypergeometric-Karsli Kantorovich Operators

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Abstract

This article introduces modified positive linear operators constructed via the Riemann–Liouville fractional integral. The proposed operators incorporate fractional parameters into the classical approximation framework, leading to a flexible structure that unifies integral-type modifications with fractional calculus. Initially, we calculate the algebraic moments and central moments of the operators, which form the basis for analyzing their approximation behavior. Utilizing these moment estimates, we demonstrate uniform convergence in $C[0,1]$ and convergence in $L^p[0,1]$ for $1 \leq p < \infty$. Quantitative estimates for the convergence rate are obtained through the first and second order moduli of smoothness, providing explicit error bounds that account for both the fractional parameter and the smoothness of the target function. Finally, a Voronovskaya-type asymptotic formula is established, delineating the exact limiting behavior of the operators and emphasizing the impact of the fractional integral component on the asymptotic expansion.

Keywords: Positive linear operators, Riemann-Liouville fractional integral, rate of convergence, hypergeometric distribution.

References:

[1] P. Dhanwani, H. Karsli and P. G. Patel, On integral generalization of hypergeometric-Karsli operators and its properties (*submitted for publication*), 2025.

[2] H. Karsli, Approximation process of a positive linear operator of hypergeometric type. Demonstr. Math. 57 (2024), no. 1, Paper No. 20230135, 12 pp.