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Cross-Approximation of 2-Variate Functions

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Abstract

Let $f(x, y)$ be a function defined on $[0, 1]^2$. The functions $f_{ab}(x, y) = f(a, y)f(x, b)$ $(a, b \in [0, 1])$ are called crosses generated by f. S. Mazur (1936) asked whether any continuous function can be uniformly approximated by linear combinations of its crosses with any accuracy. A. Grothendieck (1955) showed that this problem is in a certain way equivalent to the famous approximation problem. Since P. Enflo (1973) solved the approximation problem negatively, the answer to Mazur's question is "no", and A. Davie (1975) constructed the corresponding example explicitly.

We present several conditions on a continuous function f that are sufficient for linear combinations of its crosses to approximate it with any accuracy. For instance, the following condition is sufficient: $\sigma_m(f) = O(m^{-1/2-\varepsilon})$ $(m \to \infty,\ \varepsilon > 0)$, where $\sigma_m(f)$ denotes the least m-term uniform deviation of f with respect to the dictionary $\{g(x)h(y) : g, h \in C[0, 1]\}$. The power $-1/2$ is sharp.

However, any function $f \in L_2([0, 1]^2)$ can be approximated in L_2-norm by linear combinations of its crosses with any accuracy.

Keywords: Approximation, 2-variate function, cross-function.

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